A blended learning course in mathematics education: a case study

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Abstract

In this paper we present the whole experience of a blended course in Geometry at university level, from the design of the distance classes to the feedback of its experimentation at the University of Salerno. The main features of distance classes have been consisted in the offering of personalised units of learning and of individual and cooperative learning activities. The work done assume the necessity of integrating research outcomes in mathematics education and e-learning technology.

Keywords: e-learning, mathematics education, cooperative learning, peer assessment

Introduction

Over the last few years some mathematics courses at the University of Salerno have been supported with e-learning platforms. Thus some web-based practices in mathematical learning and teaching have been experienced in blended courses, that mix face-to-face classes and distance mathematics instruction/learning. The platform used is IWT (Intelligent Web Teacher). It is equipped with features of Learning Content Management System (LCMS), adaptive learning system and allows the definition of personalized and collaborative teaching/learning experiences by means of the explicit representation of the knowledge and the use of techniques and tools of Web 2.0.

The organization of the mathematics courses has been consisted in traditional classes, supported by distance classes. These latter have been organised into personalised courses, automatically generated by IWT and further personalised by the students themselves, and teacher-driven cooperative or individual learning activities.

The work presented in this paper fits with the theoretical framework of integrating research outcomes in mathematics education and e-learning technology in (Albano & Ferrari, 2008) and would be a concrete case of study. Research on mathematics education has widely shown the complexity of teaching and learning processes, and thus the inadequacy of one-dimensional models. In particular any model for mathematics education has to consider that students' performances are affected by factors belonging to at least three different levels: the cognitive level, the meta-cognitive level, and the non-cognitive level, which involves beliefs, emotions and attitudes, and all affective aspects, which are most often critical in shaping learners' decisions and performances. E-learning can impact on all of these aspects.

In the following we want to present the whole experience of a blended course of "Geometry" for the first year Engineering students at the University of Salerno, that is:

- all the underlying work needed to exploit the feature of IWT to offer personalised units of learning, which has been consisted in the explicit representation of the knowledge

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domain "Geometry" and in the creation of various learning objects (LOs) according to research in mathematics education outcomes;

- the setting of various learning activities, teacher-driven, performed in distance modality using some tools in IWT, and the related assessment;
- a qualitative feedback arisen from the answers to a questionnaire submitted to the students after the exam sessions.

A blended course: the methodology

In the following we are going to describe the methodology used to set a blended course of "Geometry". The course was devoted to students attending the first year of Ingegneria Edile-Architettura (engineering) and foresaw 12 week of traditional classes, each of them consists of 5 hours of face-to-face lessons. Besides, distance classes on the platform IWT have been foresaw. The number of the students attended the course is 100.

A dedicated area, with restricted access, called "Classe 2009-10 - Geometria - Ing. Edile Architettura", has been set. Once the student has been registered by the administrator, he/she can access to various sections:

a) A bacheca, for notices of general interest;
b) A personalised course, automatically generated by IWT or adjusted by the student himself/herself;
c) Tools for individual work (e.g. task);
d) Tools for cooperative work (e.g. wiki);
e) Tools for communication (e.g. mailing facilities);
f) Shared area, where everyone can pose own material he/she wants to share with the other students.

The traditional classes have been supported by some didactical material and learning activities on the platform. The students could attend their personalised course on IWT, engage in the learning activities (individual or cooperative) proposed by the teacher and take advantages from didactical resources such as previous examination tasks with solutions or summing-up exercises. In the above subsection, we will see in more details the above items b), c) and d).

The personalised course

One of the main feature of the platform IWT is the presence of three models (Didactic, Student, Knowledge) allowing the student to reach the defined didactical objectives delivering a personalised course with respect to his/her specific needs, previous own knowledge, preferred learning styles, didactical model more suitable to the knowledge at stake and to the mental model (then engagement) of the learner.

Thanks to such models, we have been able to explicitly represent the knowledge domain of Geometry (linear algebra and analytic geometry) by means of an e-learning ontology, that is a graph, where the nodes are the elementary concepts of the knowledge domain (e.g. rank, linear independent vectors, null space, eigenvalues, etc.) and the edges are mainly related to two order relations called "Is Required By" (pre-requisite) and "Suggested Order", and a decomposition relation called "Has Part" (for further details, see Albano & Maresca, 2010).

The Student Model allows to manage a learner profile, which stores information on learner preferences and needs (e.g. media, level of interactivity, level of difficulty, etc.) and cognitive state (that is, concepts of a knowledge domain which have been already learnt).

The first ones are collected by means of a questionnaire, the latter are collected by pre- and *in itinere* tests. Both are updates during the learning process through inferences by IWT according to *in itinere* tests' outcomes.

Moreover a repository of various learning objects have been created and each learning object (LO) has been annotated with metadata. This latter allows to associate the LO to the one or more nodes of the ontology, and contains further parameters, educational (e.g. level of difficulty, semantic density, didactical approach, etc.) and technical (e.g. type of media, time, etc.).

In order to offer more chance of personalization and to exploit the outcomes of mathematics education research, particular attention has been devoted to the creation of the LOs (Albano & Maresca, 2010). The tools offered by the platform have allowed to enlarge the classic proposal of didactic transposition (Chevallard, 1995) and to improve the didactical engineering (Artigue, 1992). Here is a brief description of the main ones and the ratio underlying their creation:

- *Hypermedia*: in the school practice it is evident the change in the students' style of studying/working, which is too often based on patterns of mnemonic learning and on a very focused study, neglecting variation and connections. As stressed by The National Council of Teachers in Mathematics (2000), the connections the student develop give them a grater mathematical power. According to this, some LOs which are a generalization of the hypermedia have been constructed. They are composed of a main HTML text with keywords. The links bring to other LOs, which differ as both typology of resources (e.g. animated slides, exercises, video, etc.) and educational parameters (didactical approach, semantic density, difficulty, level of interactivity, etc.). The links in the main text have been designed in order to allow the students to make connections among different topics of the mathematical knowledge, and in particular of the geometry one; to see the same concept from different viewpoint (e.g. geometrical meaning of an algebraic concept such as the determinant of a matrix); to deepen historical or motivational references; to explicit technical details (e.g. in a proof); to use various semiotic representations and their coordination (Duval, 2006);
- *Structured video:* according to Rav (1999), the whole mathematical know-how is plunged in the proofs, which contains all the mathematical methodologies, concepts, strategies for problem solving, connections among theories and so on. Based on the Rav thought, some reflections have guided the creation of suitable learning objects regarding proofs. Our starting remark is that in general a proof is not a whole inseparable text, but it is possible to single out a structure composed by several autonomous blocks, which have a proper meaning and a specific role within the proving path (e.g. sub-goals). Each of such blocks can be considered as a module which it is possible to refer to in a concise manner or in wide manner depending on the advisability. The composition of more modules leads to the construction of new knowledge, that is it allows to prove the thesis of the theorem at stake. In the above framework, some learning objects consisting in structured video, realised with a multimedia blackboard, have been designed and implemented. The videos reproduce something like a face-to-face lecture, focused on the written steps and their audio comments. Various colours have been used to address attention balancing. Pieces of previous knowledge (even in a different digital format) can be stored in other pages of the blackboard and then suitably recalled. Moreover, the videos have been split into more pieces corresponding to an educational splitting of the proof, that are the modules;

- *Static and dynamic exercises*: in order to cover the knowledge domain with problem solving skills, attention has been paid to offer learning objects on basic solving techniques. Thus two type of exercises have been implemented: a static one and a dynamic one. The first one consists in a solving model in plain text for various exercises, supplied with many comments and theoretical recalls in order to contrast the mnemonic acquisition of some procedures usually applied automatically from the students, without a previous analysis of the exercise at stake. Dynamic exercises have also been designed and implemented, using Mathematica and WebMathematica in order to create suitable algorithms generating on the fly infinity (and always different) exercises. All the algorithms are based on the divide and conquer strategy, splitting each exercise into one or more elementary steps; that is the student is guided to the solution facing easier sub-problems. At each elementary step a hint is given and an interaction is required, so that students have to give an answer to the current sub-problem. An automatic evaluation of the correctness of the given answer is done, using Mathematica. The algorithms have been suitably thought in order to recognize and distinguish errors of a (most probably) theoretical character (e.g. logical inconsistencies) and computational errors. Correspondingly, a different warning message is generated, suggesting the most likely nature of the error and suitable means of correcting it;
- *Animated slides*: they are particular meaningful when some figures comes into play. The construction of a figure is often the first and the key task to correctly solve a problem. To this aim, the conversion between verbal description and figural representation is crucial. Ferrari (2004) note that a large share of students' failures can be ascribed to linguistic issues. The animation and the synchronization between the textual description and the corresponding graphical representation allow to guide the student in such conversion. Also in this case, the animation has been designed according to some suitable elementary steps. The main topics treated in this way concern the analytic geometry. Here the conversion among verbal, graphic and symbolic representations has been treated by suitable animations, which allow to see *step by step* for instance the construction of the equations of the line or the plane in two and three dimension through a continuous migration from the graphical situation to the verbal description and to the algebraic formula. This way the learner experiences the genesis of the known equation of the line and at the same time gains experience in the coordination of different semiotic systems. The latter is a worthwhile learning activity, as such coordination is not spontaneous and it is the key of comprehension in mathematics (Duval, 2006);
- *Quiz for assessment*: a large database of questions has been created, exploiting all the admissible formats (multiple choice, true/false, matching, fill-in, cloze-procedure, short answer, numerical answer). Particular attention has been paid to the use of multiple semiotic representations and their coordination. Moreover dynamic tests using Mathematica and WebMathematica have been set, similar to the dynamic exercise previously described.

Coming back to the automatic generation of a personalized course, we (as teacher) have defined the general parameters of the Geometry course: we have chosen on the ontology the nodes representing the target concepts of the course and then we have chosen to set some milestones (how many intermediate tests and their typology the course had to be equipped with). When the student accesses for the first time to the course, IWT generates the corresponding personalized course, as described in the following: by means of the ontology, IWT creates the learning path, that is the list of the concepts needed to reach the target concepts; by means of the information stored in the learner profile (due to the Student Model), IWT adjusts such path, according to the cognitive state (knowledge already acquired) of the student; finally IWT associates to the personalized learning path the most suitable LOs according to the learner preferences (stored in the learner profile) and so the personalized course has been set (Albano et al, 2007; Gaeta et al., 2009). After the milestones, IWT proposes remedial work, if necessary. Further personalization can be done by the students themselves, allowing a non linear-approach where the learner has the liberty to explore and drive his/her own learning process (Balacheff, 2000; Maragliano, 2000), if he/she wants. Thus in IWT he/she can access alternative resources associated to each concept of the learning path, and he/she also can edit such resources realizing his/her own new resources.

Individual work

During a first half of the face-to-face classes, which have been consisted in 6 weeks, some individual homeworks have been set on the platform, using the tool "Task". It allows to set some problems with open answer, and fix some deadline to deliver the work done. We have posed six problems, with a deadline of a week for each one. Such problems aimed to engage the students in an active participation to the course and moreover they have been designed to give guidelines on the habits of studying. In fact the main design principles used are: a) Problems on theoretical questions which require that the students construct and explicit a brief reasoning; b) Problems on solving procedures which require that the students make considerations on computational efficiency or on the degrees of liberty which have impact on the subsequent solution; c) Problems on proofs which require rewriting and reworking.

There have been two kinds of evaluation of such homework. At the beginning, we have choose to examine the submitted answers of each student, then to point out the mistakes, without correcting them, to pose suitable questions with the aim of fostering student's thinking over, understanding and correct her/his mistakes by herself/himself, and to require a new submission. Later, a self-evaluation procedure has been adopted: the teacher makes available a solution sketch (more or less detailed) and the students are invited to compare it with their own product. This fosters the students to think over their products, their correctness, the probable equivalence between their own and the teacher's solving strategies, etc.

Cooperative work

In the second half of the face-to-face classes, some cooperative homeworks have been set on the platform, using the tools "Wiki" and "Forum". Some worked-out problems are made available in the platform and at first students are required to *individuate and write all the definitions, properties and theorems which get involved in procedures, indicating where they are used too*. This activities aimed to change the students' attitude towards the problems, that is more and more mechanical, leading to heavy mistakes deriving from the wrong assumption that solving procedures are disconnected by theoretical results. Afterwards, there have been further requirements: a) for those problems allowing more solving procedures, the students have been required *to apply at least one alternative procedure*; b) the teacher points out the incorrect sentences, distinguishing among wrong concepts and unclear and incomplete answers; the students are required to explain why the sentences are incorrect, providing counter-examples if possible, then to write down the corresponding correct answers; c) some problems differ just in the way the data are represented, so the resolution procedures have to differ as well; the students are required to *individuate and write the various types of problems and for each of them to sketch the solution as sequence of actions to perform*.

The evaluation phase has been cooperative too. The teacher just individuates and points out in a forum the mistakes, so the learning community is required to understand and correct them.

A qualitative feedback

After the examination sessions, a questionnaire has been submitted to the students, in order to investigate their feedback on the attended blended course. We have collected 75 questionnaire, corresponding to almost all the students who attended the blended Geometry course. Let us examine in more details the questions posed and the related answers.

First of all, we were interested to inquire which ones of the chances offered in the platform the students have been thought more useful for their training. The students have been required to give a grade between 0 and 5, according to the following legenda: 0 means that the LO or the activity has been of no help and pleasure; 3 means that it has been useful; 5 means that it has been much useful and much interesting. The graph in Figure 1 indicates the arithmetic mean of the collected answers.

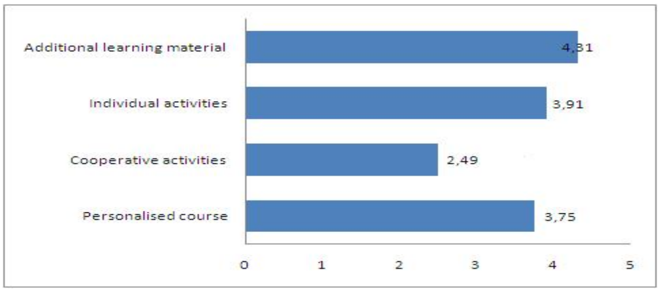

Figure 1. Arithmetic mean of the collected grades

As you can see, the cooperative activities seems to be the less useful. Actually, from further open questions, we can assume that such data is due to the concrete difficult in using cooperative tools and in editing mathematical formulas within these tools.

We have investigated on the impact of the distance classes on the students' learning outcomes and on their attitude in facing the exam. Thus, we have posed the following two questions: 1) *Do you think that the activities developed have affected your learning outcomes? Which way?* 2) *Do you think that the use of the platform have affected your attitude in facing the exam? (that is, it makes you surer or more dubious? Why? Did it give you more knowledge? Did it give you a method of studying?*

The Figure 2 shows the graph of the distribution of the answers and the evidence of the benefits.

The Tables 3 and 4 show the most common answers respectively to the open questions 1 and 2.

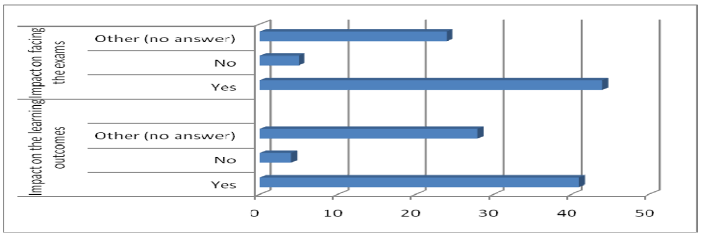

Figure 2. Distribution of the impact of the distance practices

Table 3. Impact on learning outcomes: which way?

Answers	Frequency	Percentage %
Continuous engagement	3	4.0
To become aroused	3	4.0
Assisted study	17	22.7
Effective study in depth	7	9.3
No answer	42	56.0

Table 4. Impact on facing the exam: which viewpoints?

Answers	Frequency	Percentage %
Continuous training	3	4.0
Rise in the sense of self-efficacy	24	32.0
More learning material	9	12.0
More knowledge	22	29.3
Valid point of reference	8	10.7
To get an expository method	3	4.0
No answer	39	52.0

It is worthwhile to note that the percentages of the item 2 and 4 of the Table 3 are very close. This confirm that the acquisition of more knowledge leads to a rise in the sense of self-efficacy.

A further question has been posed in order to investigate what kind of impact the activities performed in the platform have had on the exam training. The graph in the Figure 5 shows the related distribution of the answers:

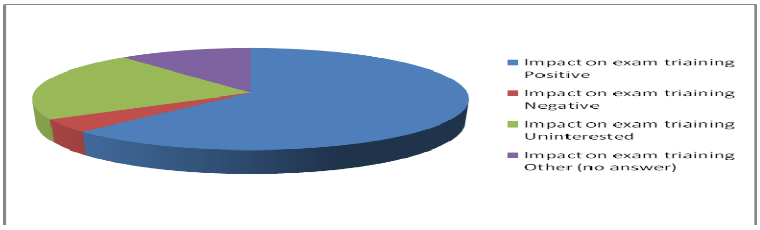

Figure 5. Distribution of the kind of impact of the distance practices

Conclusions

In this paper we have presented the whole experience of a blended course in Geometry at university level, from the design of the distance classes to the feedback of its experimentation at the University of Salerno. The distance classes have been implemented in the e-learning platform IWT, which offers facilities for personalised units of learning and for individual and cooperative work. The first ones are due to the chance of an explicit representation of the knowledge domain and of storing educational and technical data in the user profile. A didactical transposition, together with the didactical engineering, has brought to the creation of a repository of various learning objects, allowing the personalisation of the learning experience. Besides, individual and cooperative learning activities have been set in the platform, together with self and peer assessment procedures.

The collected feedback shows the usefulness and the pleasure of the students. Moreover we note the positive impact on the sense of self-efficacy, which is a key node in being successful in mathematics.

References

Albano, G., Gaeta, M., Ritrovato, P. (2007). IWT: an innovative solution for AGS e-Learning model. *International Journal of Knowledge and Learning*, 3(2/3), 209-224.

Albano, G., Ferrari, P.L. (2008). Integrating technology and research in mathematics education: the case of e-learning. In Garcia Peñalvo (ed.), *Advances in E-Learning: Experiences and Methodologies* (pp. 132-148).

Albano, G., Maresca, G. (2010). Personalised e-learning process: the case of Geometry in IWT. *2nd International Conference on Computer Supported Education*. Valencia, Spain.

Artigue M. (1992). Didactic engineering. In: R. Douady & A. Mercier (eds.). Research in didactique of mathematics: Selected papers (Special issue). *Recherches en didactique des mathématiques*. 12, 41-65.

Balacheff N. (2000). Teaching, an emergent property of eLearning environments. *The information society for all*. Retrieved 10 June 2010 from http://www-didactique.imag.fr/Balacheff/TextesDivers/IST2000.html

Chevallard Y. (1985). *La transposition didactique. Du savoir savant au savoir enseigné*. Grenoble: La Pensée Sauvage.

Duval, R. (2006). The cognitive analysis of problems of comprehension in the learning of mathematics. *Educational Studies in Mathematics*, 61(1), 103-131.

Ferrari, P.L. (2004). Mathematical language and advanced mathematics learning. In M. Johnsen Høines & F. Berit (eds.), *Proc. of the 28th Conf. of the International Group for the Psychology of Mathematics Education* (pp. 383-390), Bergen.

Gaeta, M., Orciuoli, F., & Ritrovato, P. (2009). Advanced Ontology management system for personalised e-Learning. Knowledge-Based Systems– *Special Issue on AI and Blended Learning*, 22, 292–301.

Maragliano R. (2000). *Nuovo manuale di didattica multimediale*. Editori Laterza

Rav, Y. (1999). Why do we prove theorems?. *Philosophia Mathematica*, 7(3), 5-41.

The National Council of Teachers in Mathematics (2000). *Principles and standards of school mathematics*. Reston (VA).

VV.AA. (2009a*). IWT documentation*, http://www.momanet.it/english/iwt_eng.html

VV.AA. (2009b). *Mathematica documentation*, http://www.wolfram.com/products/mathematica

VV.AA. (2009c). *WebMathematica docs*, http://www.wolfram.com/products/webmathematica